A Collision Avoidance Algorithm for Optimization-based Human Motion Prediction Based on Perceived Risk

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Abstract
Humans are able to plan movements around obstacles in a very efficient manner that almost seems effortless to the person. In digital human modeling (DHM), specifically optimization-based posture and motion prediction, the ability to computationally determine joint angle histories that effectively create motion around obstacles has been developed and employed widely. However, these methods are largely based on detecting spatial interference between objects and finding the optimal path with respect to a biomechanical cost function in an optimization problem to avoid the spatial interference. Although these methods result in a feasible path, they often times do not result in a realistic path, due to the lack of modeling for the cognitive bias that exists in real humans’ path planning in order to minimize a perceived risk. This cognitive bias is the tendency of people to grossly overestimate the low-probability of a collision between their hand and the obstacle. In turn, it results in serious sub-optimal performance, which has been documented by experimental studies in the literature. Therefore, a question arises: is it possible to account for the intentional sub-optimal human performance in a prediction model that optimizes performance? The world of cognitive science has been able to show that many biological tasks can be formulated in statistical terms, including path planning, which can account for cognitive bias. The perceived risk is formulated in terms of an economic loss or gain. This paper formulates the optimization based-motion prediction for a simple reach task around an obstacle in terms of statistical decision theory, where expected economic gains are optimized as well as biomechanical cost. The prediction models are developed in Matlab and solved using the optimization toolbox. The models are compared against motion capture data collected by a Motion Analysis System in which 6 subjects completed the simple reach task. Each subject completed ten trials in order to determine any effects due to learning as the subject became more experienced with the task.

Keywords: Optimization based-motion prediction, collision avoidance, cognitive modeling.

1. Introduction

In every movement planned and completed by people in their everyday lives, they are confronted with the need to avoid obstacles. Whether the obstacle is a glass of water sitting near an object they intend to grab, other people walking in the same path as they are, or their own bodies, people need to negotiate the obstacles in order to complete their intended tasks. People seem to be able to complete these tasks with little difficulty, and the cognitive complexity of the task is taken for granted.

The employment of collision avoidance algorithms in digital human modeling (DHM) is not a new topic. In fact, studies can be found in the literature detailing collision avoidance algorithms with specific application to DHM and optimization-based posture/motion prediction (Howard et al. 2010; Howard and Yang 2011).

Most collision avoidance algorithms applied in the literature consist of populating objects in the environment with spheres or other geometrical shapes, including the object to be avoided and also the body to be articulated, in this case the human body. Spheres are mostly used due to their geometric uniformity. Figure 2 is a visual representation of this.
In order to realize the collision avoidance, the geometrical objects are constrained within the models such that these geometries do not intersect. In the case of spheres, they are simply constrained such that the separation between sphere centers is greater than the sum of their respective radii (Howard et al. 2010; Howard and Yang 2011).

This type of implementation in optimization-based posture and motion prediction is most common. However the problem with this approach is that the cost functions of the optimization problems are typically and biomechanically driven. This means that if it is biomechanically cost efficient, the solution of the optimization problem could be that the trajectory in Figure 3 would be such that spheres touch but do not intersect. Since the goal of optimization-based posture and motion prediction is to predict realistic postures and motions, this result is unacceptable. There is a need for a collision avoidance model in which the realistic behavior of human movement is captured through cognitive modeling.

A lot of work has been done in the world of statistical decision theory to prove that human movement is in at least some part, based on perceived risk. The goal of the research in this field is to model this perceived risk as a decision among lotteries, a form of statistical decision theory (Attneave 1953; Lichtenstein et al. 1978; Kahneman and Tversky 1979; 2000; Tversky and Kahneman 1992; Sedlmeier et al. 1998; Trommershäuser et al. 2003a; 2003b; 2005; 2006; 2008; Laurence et al. 2007). In this application, it is proven that a decision among lotteries is the mathematical equivalent to a decision under risk.

The platform of statistical decision theory is a good construct for which a new collision avoidance algorithm is developed for use within optimization-based motion and posture prediction. This would allow for the modeling of the cognitive bias and sub-optimal performance (Trommershäuser et al. 2008) that is observed in human movement.

This paper formulates a new collision avoidance algorithm for use in optimization-based motion prediction that takes into account the perceived risk of the task. The formulation is based on experiments in which a simple reaching task around an obstacle was performed. The experiments included 6 subjects, 10 runs each. The subjects’ motion was recorded using a Motion Analysis motion capture system. The results from the experiments are used to derive and estimated gain function that formulates the collision avoidance algorithm as decision among lotteries, the mathematical equivalent to decision making under risk (Trommershäuser et al. 2008). The estimated gain function is used in a nested optimization algorithm in order to determine the optimal clearance around an obstacle based on the estimated gain function.

This paper is organized as follows. Section 2 gives a brief background on optimization-based motion prediction, the framework in which the new collision avoidance methodology is applied. Section 3 details the experimental methods performed in the study. The formulation of the collision avoidance algorithm is presented in Section 4. The results are described in Section 5. Then a discussion of the results and corresponding conclusions are offered in Sections 6 and 7.

2. Optimization-based Motion Prediction

2.1. Human Model

Generally, the human body can be modeled as a mechanism with revolute joints modeling the musculoskeletal joints and rigid links modeling bones (Howard, Yang et al. 2010). Many variations of digital human models have been presented in the literature; however most kinematic human models are based on the Denavit-Hartenberg method (Denavit and Hartenberg 1955). This method is used due to the computational efficiency and lack of singularities that plague other kinematic modeling methods such as the Euler angles.

The digital human model used in this study has 56 degrees of freedom (DOF). These include 13 in the spine, 9 in each arm, 7 in the head and neck, 7 in each foot, and 6 global DOFs. Fig. 1 is a visual representation of the human model used in this study. In the figure, the green cylinders represent the revolute joints.

![Figure 1: Digital human model](image)

It is important to note that although the model appears to be female, the kinematics and body segment properties of the model are un-related to the visualization of the model. More information
on this can be found in previous work of the authors (Howard and Yang 2011; Howard and Yang 2012).

2.2. Optimization

First, the general optimization problem is presented

\[ \text{Find: } [\mathbf{q}(t) \quad \dot{\mathbf{q}}(t) \quad \ddot{\mathbf{q}}(t)] \]
\[ \text{Minimize: Cost Function} \]
\[ \text{Subject to: Constraints} \]

In this case, the object of the optimization is to find the joint angles, velocities, and accelerations over time. The design variables of this optimization are the control points of uniform cubic B-splines. These are then mapped into joint space to determine the joint angles, velocities and accelerations (Ozsoy et al. 2011). This determines the unique path of the end effector (Ozsoy et al. 2011; Yang et al. 2009; Zou and Yang 2013).

The optimization finds these by minimizing the sum of the joint torque squared over time. The torque is normalized by its limits such that the joints that are physiologically able to accommodate larger torques are optimized to do so. The formulation of the joint torque calculation will not be presented in this paper for the purposes of brevity, but can be found in the literature (Howard et al. 2012).

The constraints of the model are the following.

- Joint limits simulating the physiological limits of the musculoskeletal joints.
- Torque limits simulating the amount of effort a human can put into a task.
- Distance constraints on the start and end points of the end-effector that ensure that the task predicted is the intended task.
- The time it takes for the end-effector to move from the start to end points.
- Collision avoidance constraints that ensure that body does not intersect the obstacle. This is discussed in detail in the next section.

The formulation of these constraints can be found in many studies in the literature (Yang et al. 2009; Howard and Yang 2011; Ozsoy et al. 2011; Zou and Yang 2013) and will be not provided here such that the focus of the paper stays on the collision avoidance algorithm.

2.3. Collision Avoidance

As discussed earlier, typical collision avoidance algorithms populate objects with geometrical shapes, mostly spheres due to the computational ease associated with them. Figure 2 details populating objects with spheres, the most common geometry used in collision avoidance models. Many times, single objects are represented by multiple spheres or geometries in order to develop a robust representation of the object.

After populating objects with spheres, the constraint used in optimization-based motion prediction that is the basis of collision avoidance can be developed. This constraint is rather simple. First the distances between each sphere is calculated, as shown in Figure 3.

In order to ensure the obstacle avoided, the distance vector between them, which will be referred to in this paper as the clearance vector, is constrained such that its magnitude is greater than or equal to the sum of the respective spheres’ radii. The equation below represents this concept.

\[ \| \mathbf{c} \| \geq \text{Radius}_1 + \text{Radius}_2 \]  

(1)

3. Experimental Methods

3.1. Subjects

Six subjects were recruited for this experiment and gave informed consent to participate in the experiments. All six subjects were males ranging in age from 20 to 29. The heights of the subjects ranged from 5’7” to 6’3”. All subjects were right handed.
3.2. Setup
Each subject sat on a typical office chair at a wooden table that was 30 inches in height and 36 inches square. A cup filled with water was placed in the center of the table. Start and end target points were placed 20 cm on either side of the cup and were marked by motion capture markers. The start target was placed on line with the cup and the end marker was placed 1 in offset to the right. Figure 4 is a schematic of the setup.

3.3. Setup
Motion capture was used to collect postural data. Eight Eagle-4 cameras (Motion Analysis®) were used where each camera has 4 mega-pixel resolution, shutter speeds from 0-2000 μs, focal lengths ranging from 18 mm to 52 mm, and a maximum of 500 frames per second. The capture area is 10 x 10 ft square with a height of 9 ft.

3.4. Procedure
Subjects were asked to sit at the front of the table and become comfortable with the setup. They were allowed to adjust the office chair to a height of their choosing and sit at a distance in which they could comfortably reach both the start and end targets. They were asked to complete the task once in order to become familiar with the task.

During the experiment, subjects were not required to complete the task in a given time. The goal was to recreate a natural environment and task completion strategy. Subjects were given a “Go” command once they were pointing at the start target, and the process was terminated after they reached the end target. This process was repeated 10 times for each subject. Subjects were only required to point at the start and end target markers. All movements were performed with their right hand.

4. New Collision Avoidance Algorithm
It has been stated in the literature that human movement planning can be formulated in terms of statistical decision theory. It can be effectively converted to a decision among lotteries which is the mathematical equivalent of decision making under risk (Trommershäuser et al. 2006; 2008). This section presents a method for converting the collision avoidance problem away from current methods into that of a risk based collision avoidance model.

4.1. Contextual Background
First, an experimental study from the literature is presented in order to give context to the formulation presented later in the paper. In literature (Trommershäuser et al. 2008) a movement task equivalent to decision making under risk was developed in experiment. In this case, subjects were required to reach out and touch a computer screen within a certain amount of time. On the computer screen there were two intersecting circles. Penalties and rewards, referred to as gains, were issued if the subjects touched the screen in the red circle only region, the intersection region, or in the green circle only region. These gains are shown below in the figure (penalties are negative; gains positive).

![Figure 5: Experimental setup on the computer screen and the associated penalties and gains](image)

The point of aim of each subject then becomes a choice among lotteries; where should the subject aim in order to maximize the gain? For example, the results below detail the expected gain associated with a particular aim point:

![Figure 6: Results from one instance of the experiment from Trommershäuser et al. (2006).](image)
The resulting expected gain from this aim point was -2.8 per trial, according to the study. This was calculated by the following:

$$EG = \sum_{i=1}^{3} G_i P(R_i \mid A)$$ (2)

Where $EG$ is the expected gain, $G_i$ is the gain for the $i^{th}$ region, $R_i$ is the $i^{th}$ region and $A$ is the point of Aim. $P(R_i \mid A)$ is the probability of hitting a given region given the aim point in Eq. (3). A gain of zero was associated with hitting outside any of the three regions. This was calculated assuming a Gaussian distribution and the standard deviations of the hit points from Figure 6.

$$P(R_i \mid A) = \frac{1}{2\pi\sigma^2} e^{-\frac{d^2}{2\sigma^2}}$$ (3)

Where $d$ is the distance between the aim point and the actual hit point and $\sigma$ is the standard deviation of the hit points.

Given that the subject has an infinite number of aim points to choose from, the complexity of the task is very high. Yet the experiment showed that subjects’ aim points became very close to maximizing this expected gain.

4.2. New Collision Avoidance Algorithm

Applying the theory from the experimental study by Trommershauser et al. (2006), the collision avoidance problem is formulated as a decision among lotteries or movement under risk. First, a schematic of a typical movement where an obstacle needs to be avoided is shown in Fig. 7.

By looking at the vector $c$, or the minimum clearance distance around the object, as the parameter for the decision among lotteries, this problem can modeled as movement under risk. Since $c$ can have an infinite number of possible values, the problem is finding the optimal value of $c$.

The first step in completing this is to have a metric in which the amount of clearance around a vector is quantified. For now, this has to be subjective. This metric is formulated in the form of mapping a gain function into the space around the object. Because the objects are estimated as spheres as discussed earlier, this function will be formulated in radial space around the obstacle. For the purposes of this paper, as a proof of feasibility of the model, the function will take the following form:

$$EG = -X \cdot P(\text{Collision} \mid c) + P(\text{Optimal} \mid c) \cdot Y - |c| \cdot Z$$ (4)

This function gives a penalty of magnitude $X$ for the probability in which the $c$ gives a collision. It gives a reward for the probability in which $c$ is optimal, based on the original optimization results, discussed in Section 2. Finally, it gives a penalty as $c$ increases in magnitude or as the trajectory gets farther away from the object. Basically, it limits how far the trajectory can be away from the obstacle. The magnitudes of penalty or gain $X$, $Y$, and $Z$ will are determined experimentally and discussed later in the paper.

The probabilities in Eq. (4) are calculated as in Eq. (3). The standard deviation used in this calculation is determined based on the experimental results. The subjectivity of these equations and methods will be discussed later in the paper.

4.3. Implementation in Optimization-based Motion Prediction

Applying this proposed algorithm to the formulations of optimization-based motion prediction is discussed in this section. First we outline the general process:

1) Predict motion based on formulation in Section 2. Record optimal trajectory.
2) Optimal trajectory is an input into a second motion prediction. It is used as an initial guess in the motion prediction optimization and as the input into the collision avoidance algorithm, which is a nested optimization algorithm.
3) Output of the second motion prediction is the final trajectory of the end-effector.
The optimization in step one is as presented in Section 2. For optimization in step 3, consider again the general optimization formulation as presented in Section 2 of this paper.

It has been presented that the collision avoidance is formulated as a constraint as in Eq. (1). This needs to be changed to reflect the new collision avoidance algorithm for the second optimization problem. Instead of having a static value constraint for the distance value, it is turned to a dynamic constraint that is found in a separate nested optimization algorithm. The following formulation details this optimization problem.

\[
\text{Find: } c \\
\text{Maximize: } EG = -X \cdot P(\text{Collision} \mid c) + P(\text{Optimal} \mid c) \cdot Y - |c| \cdot Z
\]

5. Results
5.1. Experimental
The results from the performed experiments are detailed in this section. First the trajectory for each run per subject is plotted along with a representation of the cup as a sphere. The trajectories can be found in Figure 2.

Figure 8: Trajectories used to complete the task for each of the six subjects. Subject 1, 2 and 3 are left to right on top, 4, 5, and 6 are left to right on bottom.

An average trajectory for the task was calculated using spatial data only, meaning the time was ignored. This trajectory can be seen below in Figure 9. It is important to note that subject six (bottom right, Figure 9) was eliminated due to the lack of smoothness at the end of the movement and was not used in the calculation of the average or used in the risk based analysis.

Figure 9: Average Trajectory across all subjects used to accomplish the task

A table was created showing the average minimum clearance that was used to avoid the object along with the standard deviation of the movement at the point of minimum clearance is shown for each subject and for the entire sample.

Table 1: The average clearance and total standard deviation between the subjects’ hand and the cup. Units are in cm.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Average Clearance (cm)</th>
<th>Std Dev (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.63282074</td>
<td>1.033292215</td>
</tr>
<tr>
<td>2</td>
<td>8.92099874</td>
<td>1.859375456</td>
</tr>
<tr>
<td>3</td>
<td>11.51047184</td>
<td>3.1181037</td>
</tr>
<tr>
<td>4</td>
<td>6.76738824</td>
<td>0.635866408</td>
</tr>
<tr>
<td>5</td>
<td>5.66049394</td>
<td>0.834074225</td>
</tr>
<tr>
<td>Average</td>
<td>7.4984347</td>
<td>2.99854117</td>
</tr>
</tbody>
</table>

The minimum clearance value is the closest that the hand comes to the center of the cup. The results of these calculations are shown in Table 1.

The time it took each subject to complete the movement is shown in the table below.

Table 2: Movement times represented in number of frames

<table>
<thead>
<tr>
<th>Subject</th>
<th>Average time (# Frames)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>281</td>
</tr>
<tr>
<td>2</td>
<td>187</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
</tr>
<tr>
<td>4</td>
<td>250</td>
</tr>
<tr>
<td>5</td>
<td>232</td>
</tr>
</tbody>
</table>

5.2. Simulation
The results of the simulation are presented along with a comparison to the experimental results from above.
First, a visualization of the trajectory calculated with traditional collision avoidance algorithms used in the optimization formulation, ignoring any perceived risk, is shown. It is plotted with the average experimental trajectories from Figure 9.

The average of all of the trajectories was used to curve fit the coefficients from equation (4) in order to arrive at viable estimated gain function. It was curve fit such that the maximum estimated gain was located at the clearance distance of the average of the trajectories. Table 2 tabulates the coefficients.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Average Experimental Clearance (cm)</th>
<th>Risk Based Collision Avoidance Clearance (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.63282074</td>
<td>5.306</td>
</tr>
<tr>
<td>2</td>
<td>8.92099874</td>
<td>6.806</td>
</tr>
<tr>
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<td>11.306</td>
</tr>
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<td>7.306</td>
</tr>
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</table>

Figure 11 shows the optimal prediction of the trajectory of the end-effector alongside the experimental trajectory.

The coefficients detailed in Table 3 were then used to predict the trajectories for the subjects individually. Fig. 12 shows the predicted trajectories by the risk based collision avoidance and the average trajectory used by each subject.

Table 4 tabulates the average experimental distances for each subject and the risk based collision avoidance predicted clearances.

<table>
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6. Discussion

The results of the experimental study paint a good picture of the cognitive bias involved in avoiding obstacles. Subjects tended to give a significant buffer zone such that the clearance of the object was guaranteed. This supports claims and other observations that were made in other studies found in the literature (Lichtenstein et al. 1978; Kahneman and Tversky 1979; 2000; Tversky and Kahneman 1992; Trommershauser et al. 2006; 2008; Laurence et al. 2007).

The comparison between the biomechanical optimal end-effector trajectory and the average is an excellent indicator of the problems facing optimization-based motion prediction. Seen in Figure 10, the optimal trajectory, though feasible as the obstacle was avoided, is nowhere near the observed tendencies from the experiments. It also
gives credence to the claim of this paper that for some motions, considerations of sub-optimal behavior and cognitive bias have place in this field of research.

The risk based collision avoidance predicted trajectory plotted alongside the average trajectories for all of the subjects should be almost the same line, as the clearance distance of the risk based collision avoidance is determined from this line. However the subtle differences are the differences between the average person completing this task, and the optimal person completing this task. Although the sub-optimal behavior due to cognitive bias is now modeled, the predicted trajectory is still biomechanically optimal within the cognitively applied constraints.

Looking at the results in Figure 12, it can be seen the estimated gain equation derived from the average trajectories of all of the subjects is a good estimator for that of each individual subject. The results displayed in this figure and the corresponding tabulated clearance distances, Table 4, is a good validator for the methodology presented in this paper.

The estimated gain function derived in this paper was a simple function used to show the methodology. It is based solely on a radial distance away from the obstacle. One of the considerations for future work would be to develop an estimated gain scalar field in which a better representation of the subject’s perceived risk.

A topic of interest is the application of the coefficients calculated in this study to other tasks. The nature of this task is almost a planar end-effector motion, although the required joint motions are three dimensional in order to realize the proper trajectory. It would be interesting to see if such a simple estimated gain function would work in more complex movements of different lengths and obstacle sizes and orientations. In other words, the results of the simulations presented in this paper are a good validator for the task presented in this paper, but it would be interesting to see how valid it would be for other situations. Further experimentation would be needed to explore this idea.

Elaborating on the previous two paragraphs the estimated gain function is based on a standard deviation of subjects’ movements. Since this model is based on experimental results, it is not subjective. However, applying this method to the theoretical person, the selection of a standard deviation for the movement would appear to be subjective. Selection of this parameter however gives the researcher the ability to predict motions for people of different skill level or coordination. Given this it is possible to vary the standard deviation in a Monte Carlo type environment that would give a range of “likely” trajectories that would result from a random population of subjects.

Focusing more on the standard deviation of the movements, one aspect to note is that the variance of trajectories seems to be mirrored about the midpoint of the movement. In other words, the middle of the movement sees the most variance and it gets smaller as one approaches the start and end positions. Linearly scaling the variance would allow for the risk based collision avoidance method to be used in situations where the obstacle is not in the center of the motion. Also, it would be interesting to study the variation of the trajectories and its correlation to the length of the movement. If there is a strong correlation (i.e. the variance is larger with longer movements), it may be possible to arrive at more meaningful estimation of the standard deviation when considering the theoretical person.

Finally it is interesting to point out that the time taken the subjects to complete the movement is generally proportional to the clearance that was used to navigate the obstacle. This provides some relationship to Fitt’s Law that says the time it takes to complete a task is directly related to its complexity (Fitts 1954). Given the assumption that the complexity of the movement increases as the clearance around the obstacle decreases, the theory developed by Fitt’s can be seen in this study.

7. Conclusion

This paper presented a method in which the cognitive bias associated with obstacle/collision avoidance in manual movements can be realized inside the framework of optimization-based motion prediction. The study included an experimental aspect where 6 subjects were required to complete a task in which an obstacle had to be avoided. The experimental results were used to derive a formula in which the perceived risk could be estimated as an economic gain/loss decision among lotteries. The simulations using the derived model had good correlation with the experimental studies as to be expected. The results of the experiments and simulations provide a strong argument that a cognitive modeling aspect should be included in the optimization-based motion prediction framework for tasks in which obstacle need to be navigated in order to predict realistic results.

Future work would include the application of this model using the derived function to simulate tasks of a different nature to test the robustness of the model. Also, a more complex function such as a scalar field could be used to model the economic
gain/loss function referred to as the estimated gain function.

References


